Variance optimization with no short positions

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The variance of a portfolio of independent, but not identically distributed, returns is optimized with a ban on short positions, in the high-dimensional limit where the number N of different assets in the portfolio and the sample size T are large with their ratio $r = N/T$ finite. To the best of our knowledge, this is the first time such a constrained portfolio optimization is carried out analytically, which is made possible by the application of methods borrowed from the statistical physics of disordered systerms. The no-short-selling constraint acts as an asymmetric l_1 regularizer, setting some of the portfolio weights to zero and keeping the out-of-sample estimator for the variance bounded, thereby avoiding the divergence present in the non-regularized case. However, the ban on short positions does not prevent the phase transition in the optimization problem, only shifts the critical point from its non-regularized value of $r = 1$ (see e.g. [1]) to 2, and changes its character: at $r = 2$ the out-of-sample variance stays finite and the in-sample variance vanishes, while a susceptibility-like quantity diverges. Numerical simulations support the analytic results for N/T.

[l] I. Varga-Haszonits, F. Caccioli, I. Kondor, J. Stat. Mech. 123404 (2016).